Homework3

PS 2010

1 Permutation and Combination

1. First question: $P(25,3) = \frac{25!}{(25-3)!} = 13800$. Second question: to calculate at least one, we could calculate the probability of no one is international student, then use 1 minus that probability. If all three positions are chosen from American students, then we have $P(15,3) = \frac{15!}{(15-3)!} = 2730$. The probability will be $\frac{2730}{13800} = 0.198$ Therefore, at least one will be $1 - 0.198 = 0.802$

2. This question is combination question because the position does not matter. Therefore, the answer is $\begin{pmatrix} 90 \\ 10 \end{pmatrix}$.

If we need 4 male and 6 female, then number of ways we can do that is $\begin{pmatrix} 20 \\ 4 \end{pmatrix} \begin{pmatrix} 70 \\ 6 \end{pmatrix}$.

Therefore the probability is $\frac{\begin{pmatrix} 20 \\ 4 \end{pmatrix}\begin{pmatrix} 70 \\ 6 \end{pmatrix}}{\begin{pmatrix} 90 \\ 10 \end{pmatrix}}$

2 Bayes Rule

1. We need to get $P(B \mid 6) = \frac{P(6 \cap B)}{P(6)} = \frac{P(B)P(6|B)}{P(6)}$. $P(6 \mid B)$ is known. This means we need to calculate $P(B)$ and $P(6)$.

From the question, we know $P(A) = \frac{6}{10} = 0.6$, $P(B) = \frac{2}{10} = 0.2$, $P(C) = \frac{2}{10} = 0.2$.

Then $P(6) = P(A)P(6 \mid A) + P(B)P(6 \mid B) + P(C)P(6 \mid C) = 0.6*0.17 + 0.2*0.8 + 0.2*0.04 = 0.268$

Finally, $P(B \mid 6) = \frac{0.16}{0.268}$

2. Define A as the event where your candidate wins the election and B as the event where the poll indicated your candidate (the Republican) will lose. Bayes' Law says

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

We can estimate these conditional and unconditional probabilities using conditional and unconditional relative frequencies. Your belief is that $P(A)$ is 72%. This is your prior. In the 25 previous, the poll indicated that the Republican candidate would lose seven times. Therefore, using relative frequency, $P(B)$ is 28% (7 out of 25). $P(B \mid A)$ is the probability that the poll indicates the Republican candidate will lose given that the Republican actually won the election. We can think of this as a "false negative:" the poll said the Republican would lose, but the poll was wrong and the Republican actually won. To estimate this conditional probability, we look only at the elections the Republican won. There are 18 elections that the Republican won. Of these 18 elections, there were three instance when the poll said the Republican would lose. Thus, our estimate for $P(B \mid A)$ is 16.7% (3 out of 18). It follows from Bayes' Law that

$$P(A \mid B) = \frac{.167(.72)}{.28} = .43$$

There is another way to calculate the P(B), which is $P(B) = P(B \mid A)P(A) + P(B \mid A^{\sim})P(A^{\sim})$. $P(A^{\sim})$ means republican lose. In our case, $P(A^{\sim}) = 1 - \frac{18}{25} = \frac{7}{25}$. Meanwhile, the poll predicts 7 times the Republican will lose, however, in this 7 times, only 4 time the poll predicts correctly, this means $P(B \mid A^{\sim}) = \frac{4}{7}$

3 Random Variable

1. (a) this question is actually asking what is $\int_0^1 \frac{2(x+2)}{5} dx$. Therefore, $\frac{2}{5}\int_0^1 x + 2dx = \frac{2}{5}(\frac{1}{2}x^2 + 2x)$, and you need to evaluate this function at 0 and 1. Finally the result will show it equals to 1.

(b) Using the same method in the above but this time evaluate the function at 1/4 and 1/2. The final answer is 19/80

2. (a) Set x is he number of defective sets, then we could write the probability function as

$f(x) = \frac{\begin{pmatrix} 2 \\ x \end{pmatrix}\begin{pmatrix} 5 \\ 3-x \end{pmatrix}}{\begin{pmatrix} 7 \\ 3 \end{pmatrix}}$. x can be 0, 1 or 2. Therefore, the probability distribution is

x	0	1	2
$f(x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$

(b) Accordingly, the $F(x)$ is

$$F(x) = \begin{cases} 0, & \text{for } x < 0 \\ \frac{2}{7}, & \text{for } 0 \le x < 1 \\ \frac{6}{7}, & \text{for } 1 \le x < 2 \\ 1, & \text{for } x \ge 2 \end{cases}$$

Finally, (c) $P(X = 1) = \frac{4}{7}$ (d) $P(0 < X \le 2) = \frac{5}{7}$

3. (a) set x be the number of erroneous entries. p(E) is the probability of making an error, p(E)=0.05. Then set p(NE) is the probability of not making error, p(NE) = 1-0.05=0.95.

If there is no error at all, that is f(0) = p(x=0) = 0.95*0.95*0.95.

If there is only one error, then it could show in the first one, or the second one, or the third one. That is f(1) = p(x=1) = (0.05*0.95*0.95) + (0.95*0.05*0.95) +(0.95*0.95*0.05) = 0.1345.

If there is two errors, then f(2) = p(x=2) = (0.05*0.05*0.95) + (0.05*0.95*0.05) + (0.95*0.05*0.05) = 0.007125

If there is three errors, then f(3) = p(x=3) = 0.05*0.05*0.05 = 0.000125

Therefore, the probability distribution is

x	0	1	2	3
$f(x)$	0.95	0.1354	0.007125	0.000125

(b) $P(x > 1) = P(x = 2) + p(x = 3) = 0.007255$

4. (a) To make f(y) as a valid density function, the integral of the density function between interval [0, 2] need to be 1. That is $\int_0^2 cydy = 1$. Solve this, you will get c = 1/2

(b) Now we could write the density function as

$$f(y) = \begin{cases} \frac{1}{2}y, & 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

We can take the integration to find F(x), that is

$$F(y) = \begin{cases} 0, & \text{for } y < 0 \\ \frac{1}{4}y^2, & \text{for } 0 \le y \le 2 \\ 1, & \text{for } y > 2 \end{cases}$$

(c) $P(1 \le Y \le 2) = F(2) - F(1) = 1 - \frac{1}{4} = \frac{3}{4}$