Homework3

PS 2010

Due: Sep 26th, 8:59:59am

Notice: The grading will mainly focus on the steps not the final results. This means even your final results are wrong but you follow the correct steps, you will be given the full score.

1 Permutation and Combination

1. In one year, three awards (research, teaching, and service) will be given to a class of 25 graduate students in our political science department. If each student can receive at most one award, how many possible selections are there?
   Continuous on the above question, if we have 10 international students, 15 Americans, what is the probability that at least one awards is given to international students

2. A study is to be conducted in a hospital to determine the attitudes of nurses toward various administrative procedures. A sample of 10 nurses is to be selected from a total of the 90 nurses employed by the hospital.
   - How many different samples of 10 nurses can be selected?
   - Twenty of the 90 nurses are male. If 10 nurses are randomly selected from those employed by the hospital, what is the probability that the sample of ten will include exactly 4 male (and 6 female) nurses?

2 Bayes Rule

1.A dishonest gambler has a box containing 10 dice which all look the same. However there are actually three types of dice.

- There are 6 dice of type A which are fair dice with $\Pr(6 \mid A) = 1/6$ (where $\Pr(6 \mid A)$ is the probability of getting a 6 in a throw of a type A die).
- There are 2 dice of type B which are biassed with $\Pr(6 \mid B) = 0.8$.
- There are 2 dice of type C which are biassed with $\Pr(6 \mid C) = 0.04$.

The gambler takes a die from the box at random and rolls it. Find the conditional probability that it is of type B given that it gives a 6.

2. You were hired to advise a political candidate running for office. Based on the records from 25 previous elections and the candidate's party affiliation, Republican, you believe she has a 72% chance of winning the election. We call this your prior belief, and it's based on a relative frequency calculation. The Republican candidate has won 18 of the 25 previous elections.

However, the most reliable poll (conducted once every election) has just been released, and it shows that a majority of the electorate plan to vote for the Democratic candidate.

In the 25 previous elections, this poll indicated that a minority of the electorate planned to vote for the Republican candidate seven times (like this election's poll), and in three of the seven cases, the Republican candidate actually won. In these three cases, the poll gave a "false negative." In the other four cases, the poll correctly predicted a Republican defeat.

How might you define the relevant events, estimate their probabilities with observed relative frequencies, and employ Bayes' Law to update your belief about your candidate's chances of winning the election? What is your updated probability of victory (your posterior belief)?

3 Random Variable

1. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

- Show that $P(0 < X < 1) = 1$
- Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

2. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel,

- Find the probability distribution of X.
- Find the cumulative distribution function of the random variable X representing the number of defectives in the above question.
- Find $P(X = 1)$
- Find $P(0 < X \leq 2)$

3. In order to verify the accuracy of their financial accounts, companies use auditors on a regular basis to verify accounting entries. The company's employees make erroneous entries 5% of the time. Suppose that an auditor randomly checks three entries.

- Find the probability distribution for Y , the number of errors detected by the auditor.
- Find the probability that the auditor will detect more than one error.

4. Suppose that Y possesses the density function

$$f(y) = \begin{cases} cy, & 0 \leq y \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

- Find the value of c that makes f (y) a probability density function.
- Find F(y).
- Use F(y) to find $P(1 \leq Y \leq 2)$..