Homework4

PS 2010

Due: Oct 17th, 8:59:59am

Notice: The grading will mainly focus on the steps not the final results. This means even your final results are wrong but you follow the correct steps, you will be given the full score.

1 Expectation and Variance

1. The probability distribution of the discrete random variable X is

$$f(x) = \left(\begin{array}{c} 3 \\ x \end{array} \right) \left(\frac{1}{4} \right)^x \left(\frac{3}{4} \right)^{3-x}, \quad x = 0, 1, 2, 3$$

- Find the mean of X

- Find the expected value of the random variable $g(X) = X^2$

- What is V(X)

2. Given

$$f(x) = \begin{cases} cx^2, & 0 \leq x \leq 2 \\ 0, & \text{elsewhere.} \end{cases}$$

- Find the value c for which f(x) is a valid density function

- Find $P(1 \leq x \leq 2)$

- Find $\mu = E(x)$ and $\sigma^2 = V(x)$

3. The profit for a new product is given by $Z = 3X - Y - 5$, where X and Y are independent random variables with $\text{Var}(X) = 1$ and $\text{Var}(Y) = 2$. What is the variance of Z ?

4. The random variables Y_1 and Y_2 are such that $E(Y_1) = 4, E(Y_2) = -1, V(Y_1) = 2$ and $V(Y_2) = 8$

- What is $\text{Cov}(Y_1, Y_1)$?

- Assuming that the means and variances are correct, as given, is it possible that $\text{Cov}(Y_1, Y_2) = 7$? [Hint: If $\text{Cov}(Y_1, Y_2) = 7$, what is the value of ρ, the coefficient of correlation?]

5. The weekly demand for a certain drink, in thousands of liters, at a chain of convenience stores is a continuous random variable $g(X) = X^2 + X - 2$, where X has the density function

$$f(x) = \begin{cases} 2(x-1), & 1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of the weekly demand for the drink.

2 Binomial Distribution

5. Suppose the probability of favoring Trump in the general population is 0.4. If 15 people randomly select from population, what is the probability that

- at least 10 people favoring Trump

- from 3 to 8 people favoring Trump

- Exactly 5 people favoring Trump

- Find the mean and variance of this distribution

3 Negative Binomial

6. Ten percent of the engines manufactured on an assembly line are defective. If engines are randomly selected one at a time and tested,

- what is the probability that the first nondefective engine will be found on the second trial?

- what is the probability that the 2 defective be found on the fifth trial? and what is the mean and variance under this condition?

4 Normal Distribution

7. Suppose that the weight of navel oranges is normally distributed with mean $\mu = 8$ ounces, and standard deviation $\sigma = 1.5$ ounces. We can write $X \sim N(8, 1.5)$. Answer the following questions:

- If you randomly select a navel orange, what is the probability that it weighs more than 11.5 ounces?

- What proportion of oranges weigh between 6.2 and 7 ounces?

- Find the value of X below which you find the lightest 80% of all the oranges.

8. Suppose the score of college entrance exam is normally distributed with mean 527 and standard deviation of 112

- What is the probability of an individual scoring above 500?

- How high must an individual score on the college entrance exam in order to score in the highest 5%?