Homework5

PS 2010

Due: Oct 31th, 8:59:59am

Notice: The grading will mainly focus on the steps not the final results. This means even your final results are wrong but you follow the correct steps, you will be given the full score.

Random Sample and Sampling Distribution

1. A random sample of 10 Pitt graduate students reported sleeping 7, 6, 8, 4, 2, 7, 6, 7, 6, 5 hours, respectively. What is the sample mean? What is the sample standard deviation?

2. A forester studying the effects of fertilization on certain pine forests in the Southeast is interested in estimating the average basal area of pine trees. In studying basal areas of similar trees for many years, he has discovered that these measurements (in square inches) are normally distributed with standard deviation approximately 4 square inches. If the forester samples n = 9 trees, find the probability that the sample mean will be within 3 square inches of the population mean.

3. An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.5 inches and if she randomly samples 100 men, find the probability that the difference between the sample mean and the true population mean will not exceed .5 inch.

4. Let $\bar{X}$ be the mean of a random sample of size 50 drawn from a population with mean 112 and standard deviation 40.

1. Find the mean and standard deviation of $\bar{X}$.
2. Find the probability that $\bar{X}$ assumes a value between 110 and 114.
3. Find the probability that $\bar{X}$ assumes a value greater than 113.

5. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a population with mean μ and variance σ^2. Consider three estimators of μ :

$$\hat{\mu}_1 = \frac{X_1 + X_2 + X_3}{3}, \quad \hat{\mu}_2 = \frac{X_1}{4} + \frac{X_2 + \cdots + X_{n-1}}{2(n-2)} + \frac{X_n}{4}, \quad \hat{\mu}_3 = \bar{X},$$

Calculate the mean squared error for each estimator. (It will be useful to first calculate their bias and variances.)

Bias and Consistency

6. Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, 1)$ and consider $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Show that $\bar{X}_n$ is a consistent estimator of θ.

Confidence Interval in Large and Small Sample

7.Consider a random sample $X_1, X_2, \ldots X_n$ from a normal distribution with mean μ and variance σ^2

1. Calculate a 95% confidence interval if, $n = 6$, $\bar{x} = 21.4$, $s^2 = 0.64$
2. Calculate a 90% confidence interval if $n = 42$, $\bar{x} = 17.2$, $s = 8$

8. In a random sample of 25 direct flights from New York to Boston by Delta Airline, the sample mean flight time was 56 minutes and the sample standard deviation was 8 minutes. Construct a 99% confidence interval for the overall mean flight time on this route. (Assume the flight times are approximately normally distributed.)

9. Just prior to an important election, in a random sample of 749 voters, 397 preferred Candidate Y over Candidate Z. Construct a 90% confidence interval for the overall proportion of voters who prefer Candidate Y over Candidate Z.